Convective dynamos: symmetries and modulation

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LESIA
Meudon

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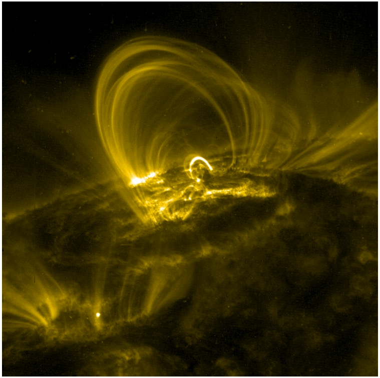
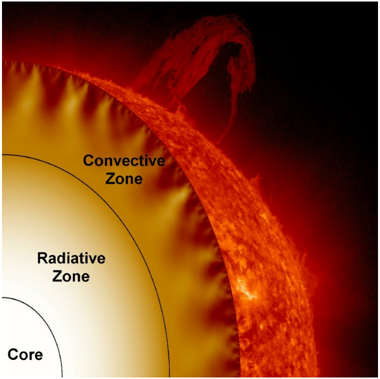
Overview

Hale (1908): "On the probable existence of a magnetic field in sun-spots"



Coronal loops

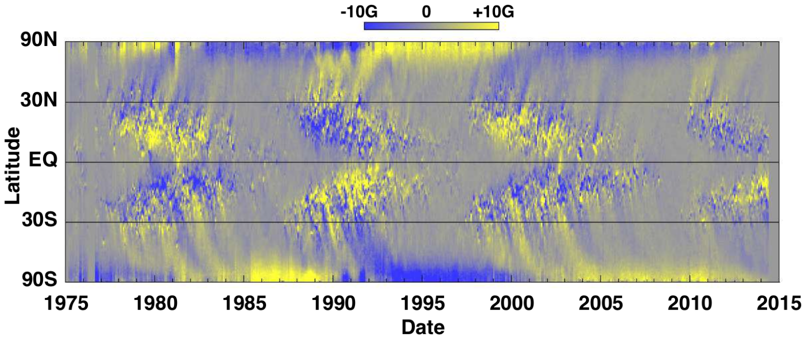
http://trace.lmsal.com



Internal structure of the Sun

http://solarscience.msfc.nasa.gov

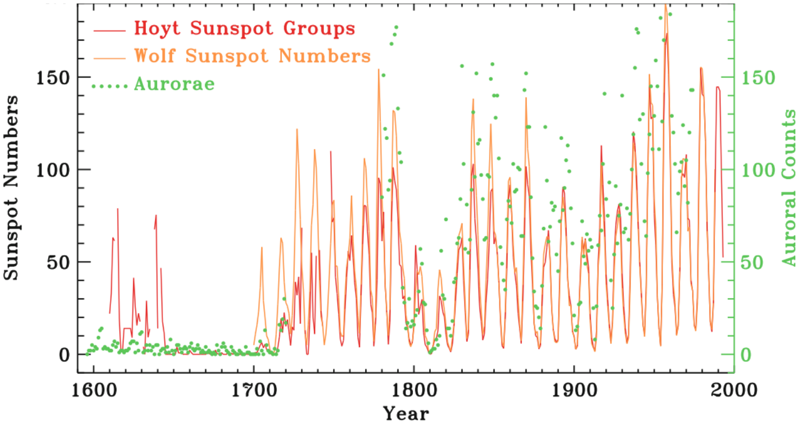
The 22 year solar cycle



Time evolution of B_r averaged in longitude at the surface of the Sun

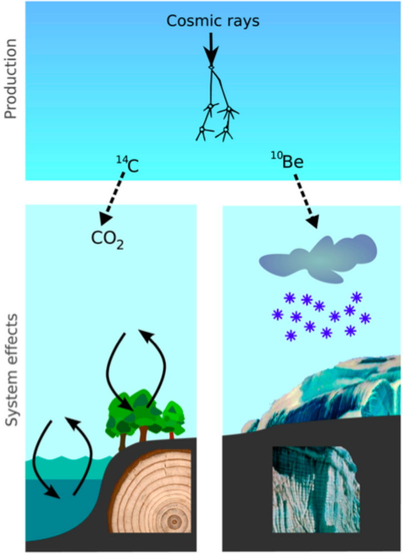
Hathaway, 2015, "The Solar Cycle", ArXiv e-prints

The Maunder Minimum (1645-1715)



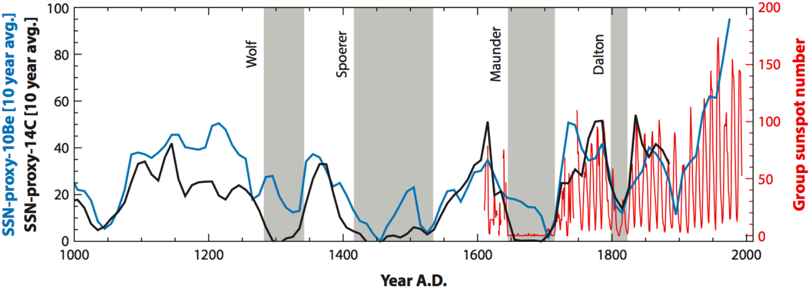
Charbonneau, 2013, *Solar and Stellar Dynamos*, Saas-Fee Advanced Course, 39, Springer-Verlag

How to go further back in time ?



Steinhilber *et al.*, 2012, Proc. Natl. Acad. Sci. USA, 109

The variability of solar activity over millennia



Time series of the group sunspot number (red) and pseudo-SSN time series constructed from cosmogenic isotopes

27 grand minima identified in the past 11 000 yr separated by aperiodic intervals of 200 yr

Charbonneau, 2014, Annu. Rev. Astron. Astrophys., 52:251–90
Usoskin, 2013, Living. Rev. Solar. Phys., 10

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Modelling the fluid flow

The Convective Approximations

Asymptotic limits of the fully compressible system that

- retain the essential physics with a minimum complexity
- filter out sound waves

The Boussinesq Approximation

$$\nabla \cdot (\mathbf{u}) = 0$$

thin layer approximation

$$L \ll H_p = \left| \frac{d \ln p}{dz} \right|^{-1}$$

The Anelastic Approximation

$$\nabla \cdot (\overline{\rho_a}\, \mathbf{u}) = 0$$

large stratified system
the lower part of which is compressed by the overlying material

The reference state

The reference state must be in quasiequilibrium:

Mechanical quasiequilibrium

hydrostatic balance

$$-\nabla P_a + \rho_a \mathbf{g} = 0 \quad (1)$$

Thermal quasiequilibrium

"well mixed" state

$$\nabla S_a = 0 \quad (2)$$

Dimensionless anelastic system

$$T_a = \zeta(r), \qquad \rho_a = \zeta^n, \qquad P_a = \zeta^{n+1}$$

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$$\begin{aligned} 0 &= \nabla \cdot \mathbf{B} \\ \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}) + \nabla^2 \mathbf{B} \end{aligned}$$

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0 &= \nabla \cdot (\rho_a \mathbf{v})
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Dimensionless anelastic system

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0 &= \nabla \cdot (\rho_a \mathbf{v}) \\
\frac{D\mathbf{v}}{Dt} &= -\frac{Pm}{E} \nabla \frac{P'}{\rho_a} + \frac{Pm^2}{Pr} Ra \frac{s}{r^2} \mathbf{e}_r - \frac{2Pm}{E} \mathbf{e}_z \times \mathbf{v} + \mathbf{F}_\nu \\
&\quad + \frac{Pm}{E\rho_a} (\nabla \times \mathbf{B}) \times \mathbf{B}
\end{aligned}$$

Dimensionless anelastic system

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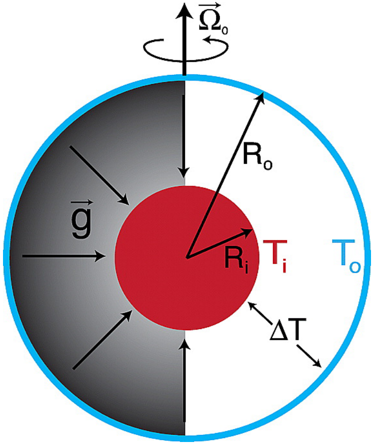
&\quad + \frac{Pm}{E\,\rho_a} (\nabla \times \mathbf{B}) \times \mathbf{B} \\
\frac{Ds}{Dt} &= (\rho_a T_a)^{-1} \frac{Pm}{Pr} \nabla \cdot (\rho_a T_a \nabla s) + \frac{Di}{T_a} \left[(E \rho_a)^{-1} (\nabla \times \mathbf{B})^2 + Q_\nu \right]
\end{aligned}$$

A simplified model for stellar convection zone



King *et al.*, 2010, GGG, Q06016

Set up

- perfect gas in a rotating spherical shell with
  - ★ constant kinematic viscosity $\nu = \mu/\rho$
  - ★ turbulent entropy diffusivity κ
  - ★ constant magnetic diffusivity η
- adiabatic reference state
- central mass distribution

Boundary conditions

- stress-free b. c. for the velocity field
- insulating b. c. for the magnetic field
- fixed entropy difference ΔS

Parameter study

Methods

- anelastic system integrated in time with a pseudo-spectral code
- focus on the interactions of modes with different

  equatorial symmetry $\begin{cases} E_b^S = \text{symmetric magnetic energy} \\ E_b^A = \text{antisymmetric magnetic energy} \end{cases}$

Control parameters

Rayleigh number	Ra	$\frac{GMd\Delta S}{\nu\kappa c_p}$	$\sim 10^6$	$\mathcal{O}(10^{20})$
Prandtl number	Pr	ν/κ	1	$\mathcal{O}(10^{-6})$
magnetic Prandtl number	Pm	ν/η	1	$\mathcal{O}(10^{-1})$
Ekman number	E	$\nu/(\Omega d^2)$	10^{-4}	$\mathcal{O}(10^{-15})$
aspect ratio	χ	r_i/r_o	0.35	0.7
polytropic index	n	$1/(\gamma-1)$	2	
number of density scale heights	N_ρ	$\ln(\rho_i/\rho_o)$	0.5	$\mathcal{O}(10)$

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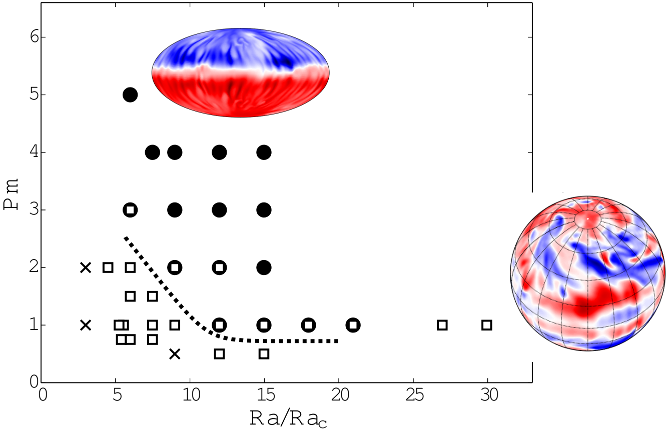
Dipolar versus oscillatory dynamos

- ● axial dipole
- □ multipole
- × no dynamo



The dynamo branches in the parameter space (Ra/Ra_c, Pm)

Raynaud *et al.*, 2015, MNRAS, 448, 2055–2065

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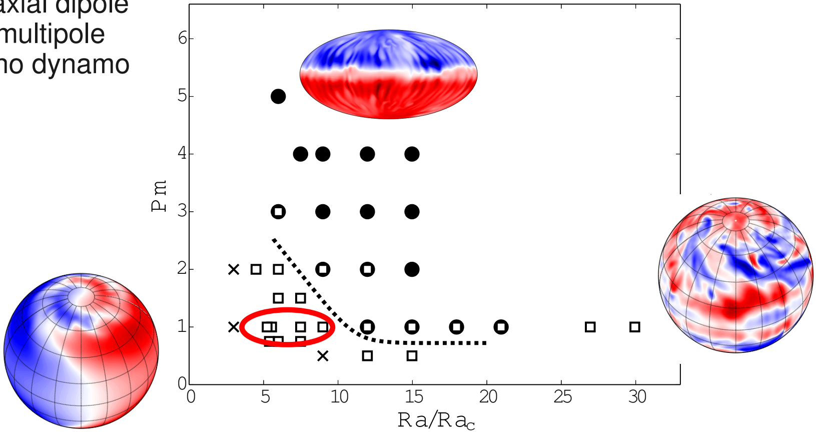

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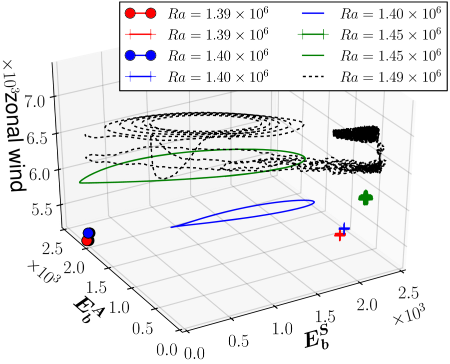
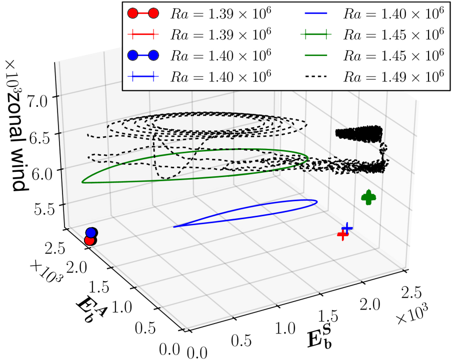
Raynaud *et al.*, 2015, MNRAS, 448, 2055–2065

Parity modulation



Phase portrait in the space (E_b^S, E_b^A, E_Z)

Parity modulation



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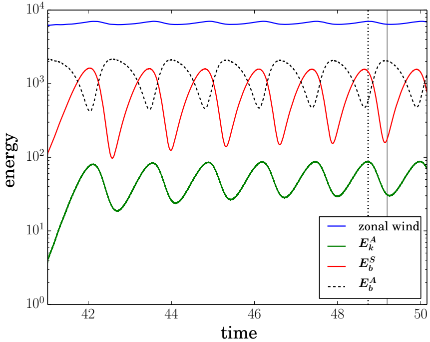
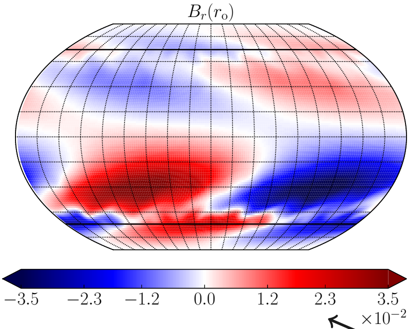


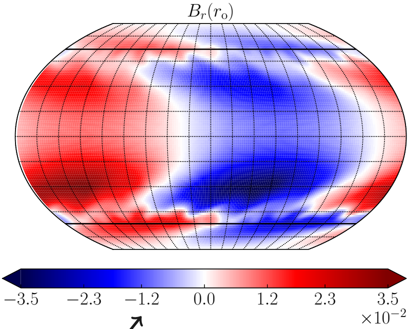
$Ra = 1.47 \times 10^6$

Type 1 modulation

- energy transfer between modes of different parity
- little change in the overall amplitude

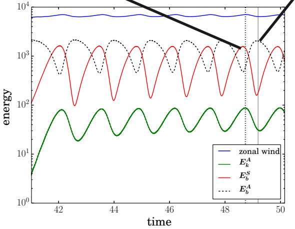
Hemispherical localisation of the magnetic field



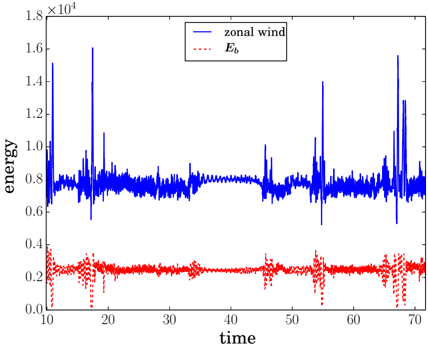


$E_b^S \gtrsim E_b^A$

$E_b^S \ll E_b^A$

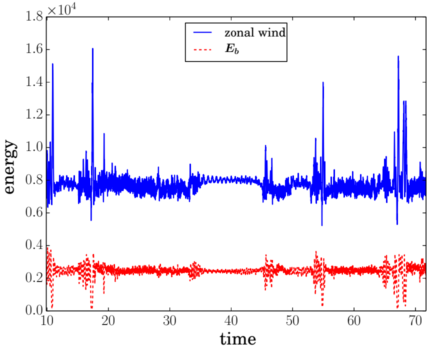


Chaotic emergence of grand minima

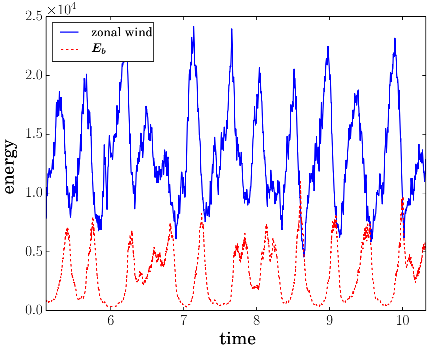


$Ra = 1.55 \times 10^6$

Chaotic emergence of grand minima



$Ra = 1.55 \times 10^6$



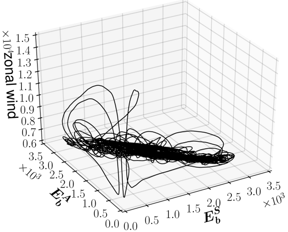
$Ra = 1.85 \times 10^6$

Type 2 modulation

- amplitude modulation via interaction with a large-scale velocity field
- no changes in symmetry

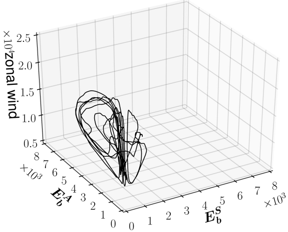
From Type 1 to Type 2 modulation

$\sim$ Type 1 modulation



$Ra = 1.55 \times 10^6$

Type 2 modulation



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Phase portraits in the space (E_b^S, E_b^A, E_Z)

From Type 1 to Type 2 modulation

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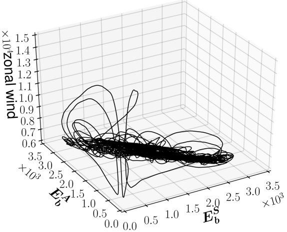
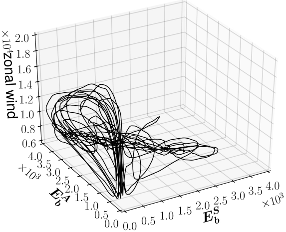


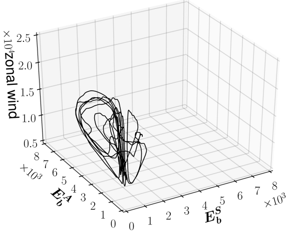
$Ra = 1.55 \times 10^6$

“super-”modulation



$Ra = 1.65 \times 10^6$

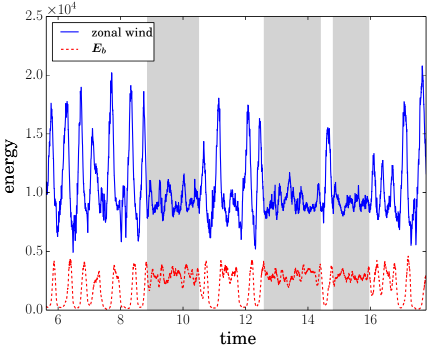
Type 2 modulation

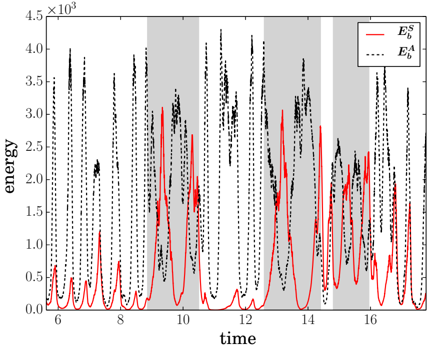


$Ra = 1.85 \times 10^6$

Phase portraits in the space (E_b^S, E_b^A, E_Z)

Supermodulation





$$Ra = 1.65 \times 10^6$$

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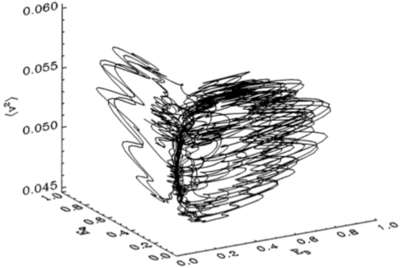
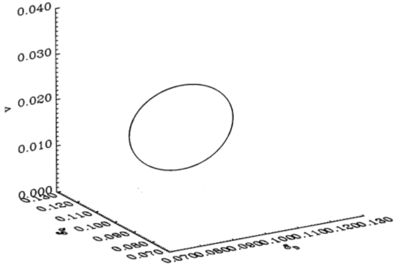
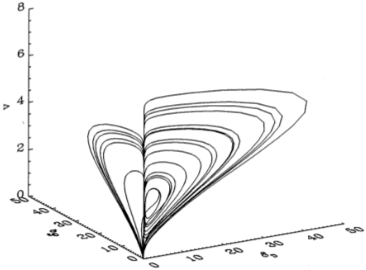
A generic dynamical behaviour

Modulations predicted by

- mean-field dynamo models
- low-order systems based on symmetry considerations

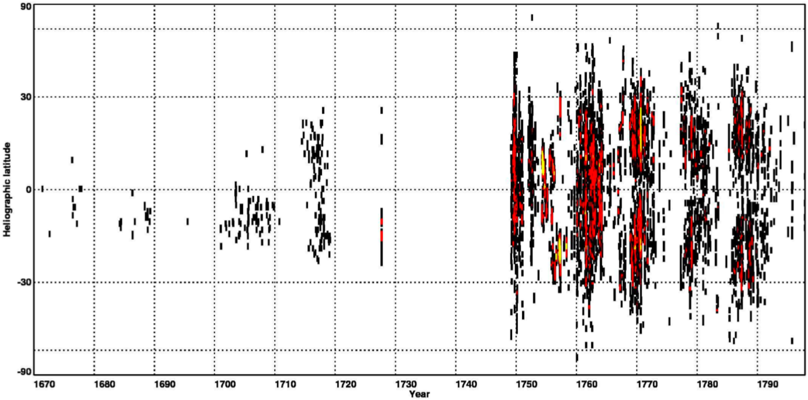






Knobloch *et al.*, 1998, MNRAS, 297, 1123–1138

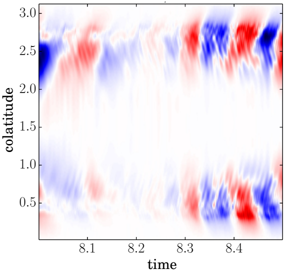
Hints for parity interactions at work in the Sun



Sunspots at the end of the Maunder Minimum

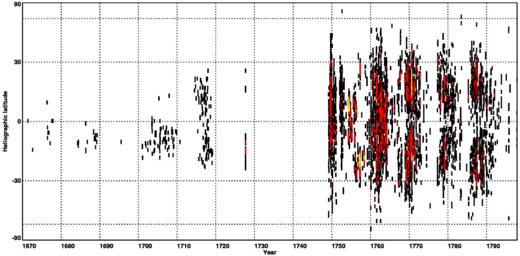
Arlt & Weiss, 2014, Space Sci. Rev., 186, 525

Hints for parity interactions at work in the Sun



Butterfly diagram
(numerical model)

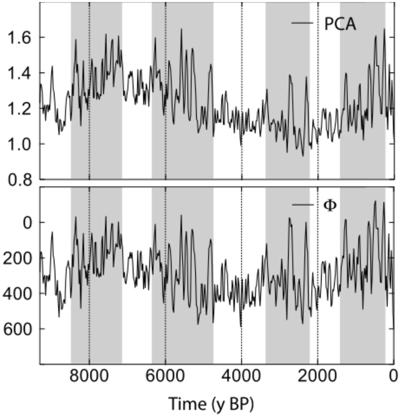
Raynaud & Tobias, 2016, JFM, 799, R6



Sunspots at the end
of the Maunder Minimum

Arlt & Weiss, 2014, Space Sci. Rev., 186, 525

Temporal variations of cosmic radiations



- PCA: combined normalized production rate of ^{10}Be and ^{14}C
- Φ: after correction for changes in the geomagnetic field

McCracken *et al.*, 2013, Solar Phys., 286, 609
Weiss & Tobias, 2016, MNRAS, 456, 2654–2661

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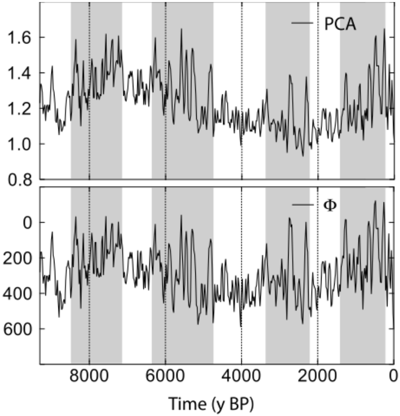
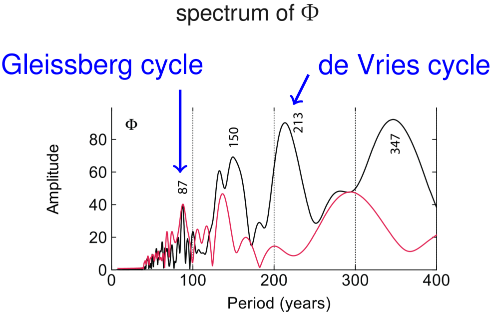




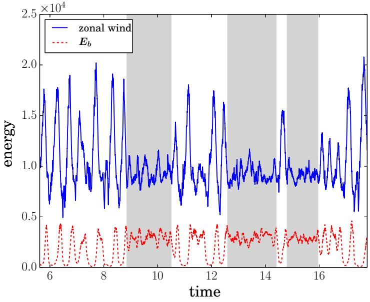
black: cluster of grand minima (shaded)

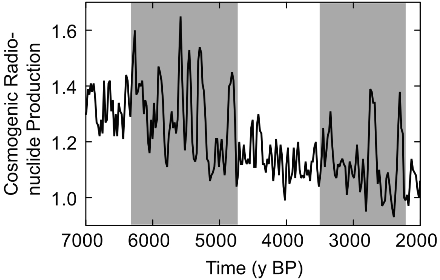
red: interval devoid of any grand minima

- PCA: combined normalized production rate of ^{10}Be and ^{14}C
- Φ: after correction for changes in the geomagnetic field

McCracken *et al.*, 2013, Solar Phys., 286, 609
Weiss & Tobias, 2016, MNRAS, 456, 2654–2661

Supermodulation of the Sun's magnetic activity





Raynaud & Tobias, 2016, JFM, 799, R6

McCracken *et al.*, 2013, Solar Phys., 286, 609

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Conclusion

Results

- 1^{st} evidence for Type 1, Type 2 and "super-"modulation in 3D direct numerical simulations
- reminiscent of the variations of the solar activity
- parity interactions may govern the long term modulation of the solar dynamo

References

- Weiss & Tobias, 2016, MNRAS, 456, 2654–2661
- Raynaud & Tobias, 2016, JFM, 799, R6

https://cv.archives-ouvertes.fr/raphael-raynaud